DOCUMENT RESUME

ED 295 978   TM 011 791

ABSTRACT

When the term "problem" is defined as a task
requiring analysis and reasoning toward a goal, it is seen that the
performance of the problem solver is narrowly delimited by the
domain, form, and complexity of the problem and the characteristics
of the solver. Tenets of a unified theory of problem solving are
discussed, concerning what the solver brings to the problem-solving
experience, what the solver does in order to solve the problem, and
the problem itself. This theory is considered along the lines of
problem solving methods used in the biological sciences. The
successful problem solver creates an internal "problem space," a
qualitative representation and redescription of the problem.
Successful problem solvers break problems into parts and apply
relevant procedures and heuristics, whether general or
domain-specific. The final phase of the solution process is
evaluation of the solution. (SLD)

TOWARD A UNIFIED THEORY OF PROBLEM SOLVING:

A VIEW FROM BIOLOGY

Mike U. Smith, Ph.D.
Mercer University School of Medicine

Paper presented at the annual meeting of the American Educational
Research Association, April 5, 1988.

1

TOWARD A UNIFIED THEORY OF PROBLEM SOLVING:
A VIEW FROM BIOLOGY

Attempting to construct a unified theory of problem solving is an awesome task and the individual who proposes to do so is certainly presumptuous and perhaps foolhardy, regardless of whether s/he has been active in the field for four years or forty. I think that all of today's participants acknowledge the enormity of the task, but we agree that there is merit is making some preliminary attempts at pulling together the fragments of understanding which have derived from our research. In other domains, such attempts have led to intense debate and disagreement, new questions and directions, and occasionally to new world views. I hope that the thoughts presented today will serve as a basis for that sort of productive exchange.

The first question which must be addressed is: Is it possible to produce a unified theory of problem solving?, i.e., Is problem solving a single construct? Much has been written lately about the domain specificity of the problem solving which occurs within various content areas, especially in the study of medical diagnosis, and I do not wish to discount these findings. Our very presence here today, however, demonstrates that we believe that there is merit in attempting to find the commonalities which overarch the differences. My guess is that the similarities among our presentations will be more striking than the differences related to our content specialties. By analogy, there is much diversity among living things, yet the concept of "life" or "living thing" has been a useful structure within which to pose questions and make observations about those things which we define as "alive". In the end, I suppose, the answer to this question will depend on how well we succeed in our attempt, i.e., how well our propositions fit into a parsimonious whole, how well they describe the way in which people solve problems, and how useful the model is in research and in the classroom.

I. Definition of terms

Researchers in the so-called hard sciences have had considerable difficulty defining such terms as "gene" and "force" and have essentially given up on trying to define "light". It seems unlikely, therefore, that researchers in the area of problem solving will ever accept some consensus definition of even the term "problem solving" itself. But, if we are to seek the commonalities in our findings, we must have common delimiters of the boundaries of the topic or at least an understanding of how our definitions vary. One case in point will demonstrate that such definition is important for communication and clearly more than an academic exercise. Recently Don Woods and I have reviewed some of each other's work to be published in the next volume of What Research Says to the Science Teacher. One basic disagreement which we had was the

role of forward chaining (sometimes called "knowledge development") vs. means-ends analysis in competent problem-solving performance. I argued that successful problem solvers most often use a forward working strategy, while Professor Woods maintained that competent problem solvers most often use means-ends analysis. This confusion was finally cleared when I realized that he does not consider the solution of "exercises" to be problem solving (Woods, 1988). We were both right; we were simply using different definitions.

Given that common definitions are important, what terms should be defined as a part of a unified theory of problem solving? Several important ones come to mind (e.g., problem solving, expert. algorithm, heuristic, etc.), but I have decided to leave that question for the discussion of the panel. I have chosen to focus on only one term, the one which I feel is most basic and which has already led to confusion in the literature-- the term "problem". Two primary issues are involved. First, must the question of whether or not a task is a problem be made with reference to the how difficult the task is for the person attempting to solve it? As indicated earlier, Woods maintains that exercises are not to be considered problems, presumably because they are less difficult. He assumes that a problem is a "stimulus situation for which an organism does not have a response", that a problem arises "when the individual cannot immediately and effectively respond to the situation" (Woods, Crow, Hoffman, and Wright, 1985). And many researchers. apparently concur. On the other hand, much problem-solving research focuses on the performance of subjects on simple exercises which in some cases are even solvable algorithmically with little or no understanding of what has been done or why it was correct.

I would propose that the word "problem" should not be defined in terms of how difficult it is for the solver. While there are clearly differences, individuals solving "exercises" or problems they have solved before still use many of the same strategies and procedures applied previously. These commonalities suggest that a similar phenomenon is taking place in the two cases. It may be more constructive to consider "completing exercises" as a subset of "solving problems". It seems to me that this issue arises from a confusion of the term "problem" as used in an academic context and in everyday language. If my wife and I are having marital difficulties, that is a "problem". If your teenage son wants to be popular but does not want to smoke pot when all his friends encourage him to, he has a "problem". The central characteristic of this kind of "problem" is the perplexity or difficulty encountered by the solver--the person doesn't immediately know what to do. I maintain that perplexity is not a necessary component of problem solving, that problem solvers often exhibit many common behaviors whether they find the task perplexing or not. If the challenge

3

4

is so perplexing as to be overwhelming, in fact, this aspect of the problem may paralyze the system so that effective problem-solving techniques cannot be appropriately summoned and applied. Problems which are perplexing, for which we have no "immediate and effective" response, do indeed call forth additional and sometimes altogether different problem-solving behaviors. On the other hand, I also find various genetics exercises to be challenging "problems" even though I have a ready store of strategies, heuristics, and algorithms with which I can "immediately and effectively" respond. That such exercises do not perplex me seems an artificial and extraneous constraint on the definition.

Second, is it a problem if it can be solved completely by memory or algorithm? For example, is "What does 2 + 2 equal?" a problem? Or, "What phenotypic ratios are expected among the offspring of a monohybrid cross between two heterozygotes?" This latter type of item is often included on typical genetics exams in a section of "problems", but I would argue that such items should not be defined as problems. How do such questions differ from any other memorization item? Would you consider "What is the capitol of the state of Louisiana?" to be a problem? Probably not. Stated in somewhat different terms, in order to be a problem, a task must require more than one step such as recall or recognition.

Perhaps again the common usage of the term causes the confusion. If a student is faced with the question on a test but doesn't know that the expected ratio is three to one or that the state capitol is Baton Rouge, then s/he does indeed have a "problem"/difficulty. The vast majority of the mental tools which can be applied to problems that cannot be solved by memorization, however, cannot be applied to ameliorate this difficulty. This is not a "problem" in what I must call the "technical" meaning of the term (vs. "common usage").

Algorithms present a slightly different concern. Landa (1972) defines an algorithm as a "completely determined . . . ready-made prescription on how to act." Lochhead and Collura (1981) add that algorithms can be "black boxes used to produce answers" with little or no understanding. By analogy, is reproducing a diagram which may have appeared in the text or on the blackboard during class solving a problem? Is identically repeating a series of steps solving a problem? I maintain that it is not. Developing the ability to reproduce a pattern and do it appropriately may indeed be learning, but performing the task is not problem solving. This distinction is based essentially on whether or not the task requires analysis and reason, which in turn require understanding of the content involved.

On the other hand, if the completion of a task requires the selection and integration of two or more algorithms, I would

4

propose that such a task may be considered a legitimate problem. Choosing the appropriate algorithms to be used, determining their sequence, and inserting the results of the first task into the second all require analysis and reason. In fact, the selection of the proper algorithm for a single algorithm task also requires a judgment and some degree of understanding of the problem. This is a form of pattern recognition and can be considered a component of problem solving. The difficulty arises when a person has only been exposed to one algorithm (e.g., the monohybrid Punnet square) in a class and then is asked to solve a problem in the class. In such a case, the student is likely to surmise that the assigned problem is to be solved using the algorithm presented. The crucial point here is that achieving a solution does not require the subject to analyze the task or reason toward its solution. (Are there other cases in which a task is a problem for certain individuals and not for others?)

To summarize the above, a problem is a task which requires analysis and reasoning toward a goal (the "solution"). This analysis and reasoning must be based on an understanding of the domain from which the task is drawn. A problem cannot be solved by recall, recognition, reproduction, or application of an algorithm alone. Whether or not a task is defined as a problem is not determined by how difficult or by how perplexing it is for the intended solver. "Problem solving", therefore becomes the process by which a system generates an acceptabl. solution to such a problem. Using these definitions, we can now seek those aspects of problem solving which are common across a variety of domains.

II.   Tenets of a unified theory of problem solving
Three basic constructs will serve as a framework for this proposal: what the solver brings to the problem-solving experience, what the solver does in order to solve the problem, and the problem itself.

A.   The problem
Research in several domains demonstrates that the performance of the problem solver is narrowly delimited by the nature of the problem being solved. Among those problem characteristics critical to determining how it will be solved are the domain from which the problem is drawn, the form in which the problem is presented (including the language used), and the complexity of the problem (Cassels and Johnstone, 1985; Gable and Sherwood, 1984; Simon and Hayes, 1976).

B.   The solver
The performance of the problem solver is also delimited by the characteristics of the solver. First, the individual brings a range of general aptitudes or capabilities to the task. The level of (Piagetian) cognitive development and field independence are two such general competencies which have been shown to

5

6

correlate with the ability to solve problems in various domains (Walker, Hendrix, and Mertens, 1979). In particular, the ability of the formal operational individual to "think about his/her own thinking", i.e., to review and analyze the process of problem solving, contributes to the ability to identify, modify, and adopt successful problem solving patterns. Similarly, field independent individuals are presumably more able to abstract the relevant information in the problem from the irrelevant "background noise" (Witkin, Oltman, Raskin, & Karp, 1971). Second the solver's performance is affected by the attitudes s/he has about the domain, about the problem-solving task, about the self, etc.

Third, the performance of the solver is determined by the relevant knowledge which s/he has and its accessibility. At least three types of knowledge have been shown to be important. First is required an adequate, well organized, and easily accessible knowledge of the relevant content domain (Chi, Feltovitch, and Glaser, 1981; Moll and Allen, 1982; Smith and Good, 1984). It is this organization and accessibility which is implied by the term "understanding." This knowledge serves as the basis upon which the solver analyzes the problem, reasons toward a solution, and assesses the appropriateness of the solution achieved. For the successful problem solver, this knowledge is free of various domain-specific misconceptions or erroneous beliefs which impede the solutions of less successful solvers. In addition to a conceptual understanding, problem solving requires procedural knowledge, both general and domain specific. This includes not only a knowledge of the different strategies, heuristics, algorithms, shortcuts, etc. which are relevant, but also the constraints under which each can be applied. Essentially, procedural knowledge consists of knowing what to do, when to do it, and how to do it (Smith, 1983). A third and related type of knowledge which bears upon the problem-solving performance is experiential knowledge. The experience of the individual at solving problems in general and at solving problems in this domain in particular often affects the choice of procedures to be applied, as well as making much of the problem-solving process tacit for that individual. Finally, problem solving is enhanced by chunking of the solver's knowledge, i.e., the individual bits of the solver's knowledge (conceptual, procedural, and experiential) are organized into larger groups or "chunks" much as a chess expert recognizes and recalls groups of chess piece positions as opposed to the positions of individual pieces (deGroot, 1965; Simon, 1981). Such chunking not only decreases the demands placed on the solver's short term memory, but also allows for the triggering of several related procedures, bits of conceptual knowledge, and memories of related experiences at one time (Chi, Feltovitch, and Glaser, 1981). Such coordination likely contributes to problem-solving success.

6

7

C.   Successful problem solving.

First, I would argue that from a research perspective expert problem solving is not identical to successful problem solving. Most experts are indeed successful problem solvers, but our research has continually identified those exceptional novice subjects who use problem-solving techniques that are very similar to those used by the experts and who can successfully solve the problems given an adequate introduction to and practice in the domain. Studying the performance of domain experts has been a fruitful research tool as essentially a easy way to identify successful problem solvers. Analysis of the performance of "successful novices", however, reveals certain differences between their problem solving and that of experts in the domain. In particular these subjects are typically more informative since much less of what they know and do is tacit information as it is in the expert subject. In addition, their performance is not confounded by the extraneous variable of experience. Given that in most academic settings (with medical diagnosis and electronic troubleshooting as notable exceptions) the educational goal is to produce successful problem solvers and not "experts" as such, I believe that our focus should turn to understanding the performance of successful solvers at a variety of levels. From this vantage point, expert problem solving is a subset of successful problem solving. It is certainly a valuable area of study, especially in fields such as medicine, but it is not the entire story.

What is it then that good problem solvers do as they work through a problem which contributes to their success and distinguishes this performance from that of unsuccessful subjects? And what are the commonalities to be found in this performance, not only in biology, but also across disciplines?

First, the problem solver creates an internal "problem space," i.e., a personal understanding of the problem (Newell and Simon, 1972). Considerable research has shown that for the successful problem solver this phase of the solution process involves the representation of the problem in terms which the solver understands, which focus on the relationship of the given problem to the solver's knowledge including similarities to previously solved problems, and which extracts the most important components of the problem in such a way as to contribute to the ease of selection and implementation of subsequent steps (i.e., decreasing the demands on short term memory) (Hinsley, Hayes, and Simon, 1977; Pople, 1977, Wortman, 1972). The recognition of problem similarities, i.e., that the given problem is an instantiation of a more general problem category or type, is particularly valuable since this recognition provides for the triggering of related chunks of content, procedural, and experiential knowledge that will guide the subsequent solution process (Chi, Feltovitch, and Glaser, 1981). This representation is often a qualitative one even when the task requires a

7

8

quantitative solution (Larkin, Heller, and Greeno, 1980).

Certain aspects of this phase have also been called redescription since the solver often abstracts salient problem features and describes them in a different manner which contributes to subsequent problem solution. The most well known example of this process is the use of free body diagrams and vectors in physics (Larkin, 1982). In algebra word problems, in fact, the redescription of the problem as an equality (formula) often constitutes the most significant part of the problem solution. Recent work with problems which approximate real world genetics laboratory data have also supported this observation (Streibel, et al, 1987). The development of a conducive initial problem space clearly depends upon the use of an adequate knowledge base.

Early on the successful problem solver may also plan the basic outlines of the general strategy or approach to be taken in the solution process. Planning may or may not occur depending on the perceived complexity of the problem. Such planning may also be tacit (not in conscious awareness) for more experienced solvers.

Next, the successful solver applies relevant problem-solving procedures to the task. The solver draws from an arsenal of two types of heuristic. The first are often called the general or weak heuristic because they are broadly applicable across a variety of domains but are typically inefficient in ensuring the rapid achievement of an appropriate solution. On the other hand, such heuristic should perhaps be considered as robust since they can be used effectively (if not efficiently) in many different areas. It is these heuristics which successful problem solvers typically apply to problems in areas in which they are not expert and which are most often the focus of present problem-solving courses. For example, all successful problem solvers tend to break a complex problem into its component parts which are then addressed individually. Other prominent examples of these techniques include trial and error and means-ends analysis.

Within areas in which an individual has more experience, the typical solver is more likely to use what Larkin (1980) calls a "knowledge development" or forward chaining approach. Instead of working backwards (means-ends analysis), such individuals tend to "work forward" from the given conditions in the problem statement, applying appropriate procedures to derive new information from them until the desired information is reached. The understanding that knowledge about the problem is being developed during the solution process is a hallmark of successful problem solving. In this way the solver modifies his/her internal problem space as more is learned about the problem. The solver must therefore also maintain a knowledge of his/her current position along the solution path.

8

9

The second type of heuristic available to the solver is domain-specific heuristics and algorithms. These techniques are generally only applicable to problems within a defined domain such as genetics and tend to be a focus of courses within these disciplines. Examples in genetics include writing down an explicit definition key for the allele symbols used, drawing all possible separate gametes, and drawing a Punnet square. These tools are very powerful in achieving quick and accurate problem solutions, but their implementation requires an adequate understanding of the content domain, of the events represented, of the criteria which must be met in order for them to be properly applied, and of how the techniques must be modified to accommodate various nuances in problems. Applying these procedures, analyzing when they are appropriate, modifying them as appropriate in the present problem, combining the results of different components, and interpreting the results into a problem solution requires logical analysis and reason (Smith and Good, 1984).

Competent problem solvers are also able to perform multi-step procedures when necessary, keeping the results of previous steps in mind (Smith, 1983). In contrast, unsuccessful problem solvers are particularly prone to attempt only one-step solution procedures whether appropriate or not. And when attempting procedures which are indeed more complex, these individuals often appear to be unable to maintain an adequate knowledge of what they have done before and to question how previous work might relate to the present position in the problem solution. Their attentional focus appears to be markedly too narrow to facilitate correct solution, perhaps related to their lack of chunking of knowledge about the domain and even about the problem solution to this point.

After the solution is generated, most unsuccessful subjects immediately stop. For most successful subjects on the other hand the final phase of the solution process is the subsequent evaluation of the solution. Again, this procedure has been noted by researchers in a variety of domains where it has been termed "solution assessment" (Reif, 1980), verification (Wallas, 1926; Schoenfeld, 1980), looking back (Polya, 1957), and checking (Smith and Good, 1984). This process can take many forms depending on the problem. It may involve a qualitative review as to the reasonability of the solution achieved, a check of the accuracy of the solver's work (mathematics, logic, etc.), and/or an assessment of the similarity of the solution to solutions achieved previously in related problems.

9

Summary

1.   Definition of the term "problem"-a task which requires
      analysis and reasoning toward a goal (the "solution"); must
      be based on an understanding of the domain from which the
      task is drawn; cannot be solved by recall, recognition,
      reproduction, or application of an algorithm alone;is not
      determined by how difficult or by perplexing the task is for
      the intended solver.
2.   The performance of the problem solver is narrowly delimited
      by the nature of the problem being solved--its domain, for
      (and language), and complexity.
3.   The performance of the problem solver is also delimited by
      the characteristics of the solver:
      general aptitudes/capabilities (e.g., cognitive development
           and field independance
      attitudes
      knowledge/understanding
           adequate, well organized/chunked, easily accessible
           types: conceptual, procedural, experiential.
4.   The successful problem solver creates an internal "problem
      space" which is a qualitative representation, redescription
      of the problem and includes categorizing the problem.
5.   Depending on the perceived complexity of the problem, the
      successful problem solver may (at least tacitly) plan the
      general strategy or approach to be taken.
6.   Successful problem solvers break problems into parts and
      perform multi-step procedures when necessary, keeping the
      results of previous steps in mind.
7.   The successful solver applies relevant problem-solving
      procedures/heuristics-two types:
      general heuristics (e.g., trial and error, means-ends
           analysis, knowledge development)
      domain-specific heuristics/algorithms.
8.   Problem solving requires logical analysis and reason.
9.   The final phase of the solution process is the evaluation of
      the solution.

Bibliography

Cassels, J. R. T., & Johnstone, A. H. (1984). The effect of language on student performance on multiple choice tests in chemistry. Journal of Chemical Education, 61, 613-615.

Chi, M.T.H., Feltovich, P.J., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. Cognitive Science, 5, 121-152.

deGroot, A. D. (1965). Thought and choice in chess. The Hague: Mouton.

Gabel, D. L., & Sherwood, R. D. (1984). Analyzing difficulties with mole-concept tasks by using familiar analog tasks. Journal of Research in Science Teaching, 21, 843-851.

Hinsley, D. A., Hayes, J. R., & Simon, H.A. (1977). From words to equations: Meaning and representation in algebra word problems. In M. A. Just & P. A. Carpenter (Eds.), Cognitive processes in comprehension. Hillsdale, NJ: Lawrence Erlbaum Associates.

Landa, L. N. (1972). Algorithmization in learning and instruction. Englewood Cliffs: Educational Technology Publications.

Lochhead J., & Collura, J. (1981). A cure for cookbook laboratories. The Physics Teacher, 19, 46-50.

Larkin, J. (1980). Teaching problem solving in physics: The psychological laboratory and the practical classroom. In D.T. Tuma & F. Reif (Eds/), Problem solving and education: Issues in teaching and research. Hillsdale, NJ: Lawrence Erlbaum Associates.

Larkin, J. H. (August, 1980). Spatial reasoning in solving physics problems (C.I.P. #434). Unpublished manuscript, Pittsburgh: Carnegie-Mellon University.

Larkin, J. H., Heller, J. I., & Greeno, J. G. (1980). Instructional implications of research on problem solving. New Directions for Teaching and Learning, 2, 51-65.

Moll, M. B., & Allen, R. D. (1982). Developing critical thinking skills in biology. Journal of College Science Teaching, 12, 95-98.

..., & Simon, H.A. (1972) Human problem solving. wood Cliffs, NJ: Prentice-Hall.

11

Polya, G. (1957). How to solve it. Garden City, NY: Doubleday.

Pople, H. E. (1977). The formation of composite hypotheses in diagnostic problem solving: An exercise in synthetic reasoning. Proceedings of the 5th International Joint Conference on Artificial Intelligence. Cambridge, Mass.

Reif, F. (1980). Theoretical and educational concerns with problem solving: Bridging the gaps with human cognitive engineering. In D. T. Tuma & R. Reif (Eds.), Problem-solving and eduction: Issues in teaching and research. Hillsdale, NJ: Lawrence Erlbaum Associates.

Schoenfeld, A. H., & Herrman, D. J. (1980). Problem perception and knowledge structure in expert and novice mathematics problem solvers. Clinton, NY: National Science Foundation. (ERIC Document Reproduction Service No. ED 200 609).

Simon, H. A. (1981). Information-processing models of cognition. Journal of the American Society for Information Science, 32, 364-377.

Simon, H.A., & Hayes, J. R. (1976). The understanding process: Problem isomorphs. Cognitive Psychology, 8, 165-190.

Simon, H. A., & Paige, J. M. (1979). Cognitive processes in solving algebra word problems. In H. A. Simon (Ed.), Models of thought. New Haven: Yale University Press.

Smith, M. U. (1983). A comparative analysis of the performance of experts and novices while solving selected classical genetics problems. (Doctoral dissertation, The Florida State University). Dissertation Abstracts International, 44, 451-A.

Smith, M.U., & Good, R. (1984). Problem solving and classical genetics: Successful versus unsuccessful performance. Journal of Research in Science Teaching, 21, 895-912.

Streibel, M. J., Stewart, J., Koedinger, K., Collins, A., and Jungck, J. R. (1987). Mendel: An intelligent computer tutoring system for genetics problem-solving, conjecturing, and understanding. Machine Mediated Learning, 2, 129-159.

Walker, R. A., Hendrix, J. R., & Mertens, T. R. (1980). Sequenced instruction in genetics and Piagetian cognitive development. American Biology Teacher, 42, 104-108.

Wallas, G. (1926). The art of thought. New York: Carcourt, 1926

12

Witkin, H. A., Oltman, P. K., Raskin, E., & Karp, S. A. (1971).
    A manual for the embedded figures tests. Palo Alto, CA:
    Consulting Psychologists Press.

Woods, D. R. (in press). Problem solving in practice. In D.
    Gabel (Ed.). What research says to the science teacher. Vol.
    IV. Washington, DC: National Science Teachers Association.

Woods, D. R., Crowe, C. M., Hoffman, T. W., & Wright, J. D.
    (1985, January). Challenges to teaching problem solving
    skills. Chem 13 News (No. 155, pp. 1-12). Waterloo,
    Ontario, Canada: University of Waterloo.
Wortman, P. M. (1972). Medical diagnosis: An information
    processing approach. Computers and Biomedical Research, 5,
    315-328.